2021

MATHEMATICS — HONOURS

Paper : CC-11

(Probability and Statistics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Each of the following questions has four possible answers of which exactly one is correct. Choose the correct alternative with proper justification (wherever applicable) : 2×10

(a) A and B are two independent events such that $P(\bar{A}) = 0{\cdot}7, P(\bar{B}) = K$ and $P\ (A \cup B) = 0{\cdot}8$. Then K is

(i) $\frac{5}{7}$ (ii) 1

(iii) $\frac{2}{7}$ (iv) $\frac{4}{7}$.

(b) The standard deviation of a uniformly distributed random variable in $0 \le x \le 1$ is

(i) $\frac{1}{\sqrt{3}}$ (ii) $\frac{1}{\sqrt{12}}$

(iii) $\frac{5}{\sqrt{3}}$ (iv) $\frac{1}{\sqrt{2}}$.

(c) If $f(x)$ is a p.d.f. then the value of c for the following function is

$$f(x) = \begin{cases} cx & \text{if } 0 \le x \le 6 \\ c(12-x) & \text{if } 6 \le x \le 10 \end{cases}$$

(i) $\frac{1}{34}$ (ii) $\frac{1}{43}$

(iii) $\frac{1}{3}$ (iv) $\frac{1}{2}$.

(d) If $\varphi(t)$ is the characteristic function of a $r.v.X$, then that of $a+bX$ will be (a and b being two const.)

(i) $e^{ita}\varphi_x(tb)$ (ii) $e^{-ita}\varphi_x(tb)$

(iii) $e^{itb}\varphi_x(ta)$ (iv) $e^{-itb}\varphi_x(ta)$.

(e) If the joint p.d.f. of (X, Y) is given by

$$f(x,y)=\begin{cases}\frac{1}{4}, & 0\le x,y\le 1\\ 0, & \text{otherwise}\end{cases}$$

then $P(X+Y\le 1)$ is equal to

(i) $\frac{1}{3}$ (ii) $\frac{1}{16}$

(iii) $\frac{1}{8}$ (iv) $\frac{1}{4}$.

(f) Let S^2 and σ^2 respectively denote the sample and population variances for a sample of size N. Then, $E(S^2)$ equals

(i) $\frac{\sigma^2}{N}$ (ii) $\sigma^2-\frac{\sigma^2}{N}$

(iii) $\sigma^2+\frac{\sigma^2}{N}$ (iv) None of these.

(g) For a normal (m, σ) population, consider a sample of size n. If $u=\sqrt{n}\frac{\bar{x}-m}{\sigma}$, and $P(-u_\in<U<u_\in)=1-\in$, $\bar{x}$ being the sample mean and U the random variable corresponding to u, then $P(U\ge u_\in)$ equals

(i) $\frac{\in}{2}$ (ii) $1-\frac{\in}{2}$

(iii) $1+\frac{\in}{2}$ (iv) None of these.

(h) The variables X and Y are connected by $2X-3Y+5=0$. The correlation coefficient between them is

(i) -1 (ii) 1

(iii) $\frac{2}{3}$ (iv) none of these.

(i) For $f(x,\theta)=\frac{1}{\theta};0\le x\le\theta$, to test the hypothesis $H_0:\theta=1$ against $H_1:\theta=2$ in the critical region $1\le x\le 1{\cdot}5$, Type I error is

(i) $0{\cdot}5$ (ii) $0{\cdot}75$

(iii) $0{\cdot}25$ (iv) $0{\cdot}15$.

(j) Consider a bivariate sample (x_i, y_i) with correlation coefficient r. The regression coefficients b_{xy} and b_{yx} are connected by

(i) $b_{xy}b_{yx} = \dfrac{1}{r^2}$ (ii) $b_{xy}b_{yx} = r^2$

(iii) $\dfrac{b_{xy}}{b_{yx}} = \dfrac{1}{r^2}$ (iv) None of these.

Unit - 1

Answer ***any two*** questions.

2. State the axioms of probability. When is a collection of events $\{A_i, i = 1,2,...,n\}$ said to be 'mutually exclusive events' and when is it said to be 'mutually independent events'? For any two events A, B show that

$$P\left(A\overline{B} + B\overline{A}\right) = P(A) + P(B) - 2P(AB).$$ 1+2+2

3. An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then one ball is taken at random from the second urn. What is the probability that the ball drawn is a white ball? 5

4. Obtain the recurrence relation

$$\mu_{K+1} = \lambda\left(K\mu_{K-1} + \frac{d\mu_K}{d\lambda}\right)$$

for the Poisson distribution with parameter λ. (symbols have their usual meanings) 5

Unit - 2

Answer ***any two*** questions.

5. If x is a $N(0, 1)$ variate then find the distribution of

(a) $Y = \dfrac{1}{2}X^2$

(b) $Y = e^X$ 3+2

6. If $f(x, y) = 3x^2 - 8xy + 6y^2$ $(0 < x < 1,\ 0 < y < 1)$, find $f_x(x \mid y)$ and $f_y(y \mid x)$ and show that X and Y are dependent. 5

7. A continuous random variable X is distributed over the interval (0, 1) with p.d.f. $ax^2 + bx$, where a and b are two constants. If $E(X) = 0{\cdot}5$ then find the values of a and b. Hence find the third central moment of the random variable X. 3+2

Unit - 3

Answer ***any one*** question.

8. If a random variable X possesses a finite second order moment about $c \in \mathbb{R}$, then show that, for any $\epsilon > 0$,

$$P(|X - c| \geq \epsilon) \leq \frac{E\left\{(X - C)^2\right\}}{\epsilon^2}$$ 5

9. State limit theorem for characteristic function. Use it to obtain Poisson distribution as a limiting case of binomial distribution. 5

Unit - 4

Answer ***any two*** questions.

10. (a) Explain sampling distribution and distribution of a sample.

(b) Distinguish between Simple Random Sampling with Replacement (SRSWR) and Simple Random Sampling without Replacement (SRSWOR). 3+2

11. Find the maximum likelihood estimate of $\lambda(\lambda > 0)$ in case of a Poisson distribution with probability mass function

$$P(X = i) = \frac{e^{-\lambda}.\lambda^i}{\underline{|i}}, \quad i = 0,1,2,3,...$$ 5

12. What is meant by the confidence interval for a parameter of a distribution? Find the confidence interval for the variance of a normal (m, σ) population on the basis of a random sample of size n and a confidence coefficient $1 - \alpha$. 5

13. The rainfall of a rainy season of a particular region in India is measured for 10 days and the measurements (in mm) are 9·4, 8·8, 10·6, 12·2, 11·8, 11·4, 9·9, 10·8, 12·1, 11·7. Compute 99% confidence interval for standard deviation of the population, assuming the population of measurements of rainfall is normal.

$$\left[\text{Given, } \chi^2_{0\cdot005,9} = 23\cdot59,\ \chi^2_{0\cdot995,9} = 1\cdot73\right]$$ 5

Unit - 5

Answer ***any two*** questions.

14. The fraction of defective items in a large lot is p. To test the null hypothesis H_0 : p = 0.2 one considers the number of defectives in a sample of 8 items. The null hypothesis is accepted if X $\leq$ 6 and rejected otherwise.

(a) What is the probability of type-I error of this test?

(b) What is the probability of type-II error corresponding to p = 0.01?

(c) Calculate the power of the test for p = 0.01. 2+2+1

15. Nine patients to whom a certain drug was administered, registered the following rise in blood pressure in mm of Hg : 3, 7, 4, –1, –3, 6, –4, 1, 5. Test the hypothesis that the drug did not raise blood pressure at 10% significance level assuming that the sample is chosen from a normal population. Given : $P(t > 1.86) = 0.05$ for eight degrees of freedom. 5

16. The regression equations for a bivariate sample are given by $x + 2y - 5 = 0$ and $2x + 3y - 8 = 0$. If $S_x^2 = 12$, calculate the values of $\bar{x}, \bar{y}, S_y$ and ρ, where symbols carry usual meanings. 5

17. Fit an exponential curve to the following data : 5

x	1·0	1·5	2	2·5
y	0·4	0·9	1·2	2

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